Suggested curriculum in Differential Geometry

Responsible: Prof. Barbara Opozda

1st year

title	kind of activity	hours/ week	hours/ year	form of crediting	credits
General School Seminar	seminar	2	60	participation	**4**
Seminar	seminar	2	60	participation	**6**
Mathematical Analysis	lectures, classes	2 2	60 30	exam	**12**
Analysis on Manifolds	lecture	2	30	exam	**4**
Differential Topology	lectures	2	30	exam	**4**
Partial Differ. Equations	lectures	2	30	exam	**4**
Riemannian Geometry	lectures	2	30	exam	**4**
Fiber Bundles	lectures	2	30	exam	**4**
Lie Groups and Representation Theory	lectures	2	30	exam	**4**
Tutorials	tutorial	2	60	as arranged with tutor	**4**

2nd year

title	kind of activity	hours/ week	hours/ year	form of crediting	credits
General School Seminar	seminar	2	60	participation	**4**
Seminar	seminar	2	60	participation	**6**
Natural Bundles	lectures	2	30	exam	**4**
Characteristic Classes	lectures	2	30	exam	**4**
Affine Geometry	lectures	2	30	exam	**4**
Riemannian Geometry and Holonomy Groups	lectures	2	30	exam	**4**
Geometry and Physics	lectures	2	30	exam	**4**
Geometric Structures on Foliated Manifolds	lectures	2	30	exam	**4**
Tutorials	tutorial	2	60	as arranged with tutor	**4**
Diploma project	individual work	10	300	diploma exam	**30**

Analysis on Manifolds. Atlases, orientability, complex structures. Tangent and cotangent bundles. Differentiable functions, differential. Submanifolds. Vector fields, Poisson bracket. Tensor bundles, tensor fields. Differential forms, wedge product. Derivatives, Lie derivative, exterior derivative. Integration of forms. Manifolds with boundary. The Stokes theorem.
Responsible: Prof. Barbara Opozda

Differential Topology. The topics discussed will include function space: strong and weak topologies, transversality and Thom-Sard's theorem, the Frobenius theorem, both for vector fields and differential forms, the basics of the foliation theory including singular foliations, the theory of differential form and the de Rham cohomology, including the Mayer-Vietoris exact sequence.
Responsible: Assoc. Prof. Robert Wolak

Riemannian Geometry. Metric tensor fields. The Levi-Civita connection. Completeness. Sectional curvature, spaces of constant curvature. Einstein manifolds. Riemannian submanifolds. Hypersurfaces, shape operator, second fundamental form. Fundamental equations, fundamental theorems. Minimal submanifolds. The Gauss-Bonnet theorem.
Responsible: Prof. Barbara Opozda

Partial Differential Equations. Well-posed problems, classical, weak and viscosity solutions. Transport equation. Second -order elliptic equations. Second order parabolic equations. Applications

of PDF to stochastic processes. Second order hyperbolic equations. Weak solutions of the boundary value problems for Poissons equations.
Responsible: Prof. Bolesław Szafirski

Fiber Bundles. Actions of Lie groups on manifolds, principal fiber bundles, examples, reduced bundles, associated bundles. Vector bundles and algebraic operations on vector bundles. Connections on principal fiber bindles, connection forms, curvature forms, structural equations and Bianchi identifies. Canonical forms on the linear frames principal bundles and torsion forms, structural equations and Bianchi identifies on the linear frame bundles. Covariant derivations, relations between torsion and curvature tensors and torsion and curvature forms.
Responsible: Prof. Jacek Gancarzewicz

Lie Groups and Representation Theory. The first part of the course starts with a definition of Lie groups and Lie algebras, Engel's theorem and Lie's theorem. A detailed discussion of low-dimensional algebras and relationships between them is provided. Representation theory of sl(2,C) follows. This is then extended to the case of classical Lie algebras: special unitary, symplectic and orthogonal algebras. In this part numerous explicit examples are provided and analysed. The second part is devoted to the classification of complex simple Lie algebras. The relationship between these algebras and their Dynkin diagrams is established. Representation rings and characters and the Weyl character formula are introduced. This is then applied to classical Lie algebras.
Responsible: Dr Piotr Kobak

Natural bundles. Natural bundles, examples, bundles of r-jets. Order of natural bundles and Palais-Chern theorem. Classifications of natural bundles. Product preserving functors, their Weil algebras, an classification. Properties of natural bundles and product preserving functors. Natural transformations, invariants. Examples of classifications of some natural transformations and invariants. Invariants on some natural bundles.
Responsible: Prof. Jacek Gancarzewicz

Characteristic Classes. The basic construction of the Chern-Weil construction of characteristic classes for principal fiber bundles using invariant polynomials and curvature forms of connections. In this way the Pontrjagin and Chern classes are introduced. It will be checked that these classes satisfy the axioms of characteristic classes. The next class to be discussed is the Euler class. Finally, we will discuss characteristic classes as obstructions to the existence of certain geometric structures. At the end secondary characteristic classes and their variations will be discussed.
Responsible: Assoc. Prof. Robert Wolak

Affine Differential Geometry. Non-degenerate hypersurfaces in R^n, classical structures. Affine immersions. Fundamental equations and fundamental theorems. The Cartan-Norden theorem. Convexity and ovaloids. Affine spheres and affine minimal hypersurfaces. Integral formulas and rigidity theorems. Affine homogeneity.
Responsible: Prof. Barbara Opozda

Riemannian geometry and holonomy groups. Prerequisites: Introduction to Differential Geometry, Lie Groups and Introduction to Representation Theory. This is an advanced lecture course in differential geometry. The aim is to use holonomy groups and their representations to provide a uniform approach to various Riemannian geometries.
The course starts with a brief recap of Riemannian manifolds, Levi-Civita connection and holonomy groups. Kaehler, special Kaehler, hyperKaehler and quaternion-Kaehler manifolds are introduced as Riemannian manifolds with holonomy contained in Lie groups U(m), SU(m), Sp(n) and Sp(n)Sp(1) respectively. Basic properties of these manifolds and their curvature are studied. The link between quaternion-Kaehler and hyperKaehler geometries via Swann bundles and twistor spaces is established. The special case of four dimensions is treated separately. Finally, two exceptional holonomy metrics: G2 and Spin(7) are considered.
Responsible: Dr Piotr Kobak

Geometry and Physics. We begin by describing a general framework for the classical gauge theories of physics and then discuss a number of specific examples. These include classical electromagnetic theory, Dirac monopoles and SU(2) Yang-Mills-Higgs theory. We study Hamiltonian and Lagrangian systems on symplectic manifolds, Minkowski spacetime, spacetime manifolds and some concrete examples (the Einstein-de Sitter spacetime, de Sitter spacetime, Einstein cylinder, Schwarzschild and Kerr spacetime).
Responsible: Dr Zdzisław Pogoda

Geometric Structures on Foliated Manifolds. The subject of the lecture course is the interplay between the structure of a foliated manifold and the existence of geometric structures adapted in some way to the foliation. A lot of attention is paid to foliations on Riemannian and Kähler manifolds (the structure theorem, basic cohomology, minimality, spectral geometry) as well as on symplectic manifolds (lagrangian foliations). A brief survey of the theory of foliations of semi-Riemannian and Lorentz manifolds and their applications (e.g. the space-time models) completes the lecture course.
Responsible: Assoc. Prof. Robert Wolak