Suggested curriculum in Singularities

Responsible: Prof. Wiesław Pawłucki

1st year

Title	kind of activity	hours/ week	hours/ year	form of crediting	credits
General School Seminar	seminar	2	60	participation	**4**
Seminar	seminar	2	60	participation	**6**
Mathematical Analysis	lectures, classes	2 2	60 30	exam	**12**
Algebraic Geometry	lecture	2	60	exam	**7**
Differential Topology	lectures	2	30	exam	**4**
Semi-algebraic geometry	lectures	2	30	participation	**3**
Commutative Algebra	lectures	2	60	exam	**7**
Singularities of Smooth Mappings	lectures	2	60	exam	**8**
Semianalytic and Subanalytic Geometry	lectures	2	30	exam	**5**
Tutorials	tutorial	2	60	as arranged with tutor	**4**

2nd year

Title	kind of activity	hours/ week	hours/ year	form of crediting	credits
General School Seminar	seminar	2	60	participation	**4**
Seminar	seminar	2	60	participation	**6**
Extension problems for Differentiable Functions	lectures	2	30	exam	**4**
O-minimal structures	lectures	2	30	exam	**5**
Characteristic Classes	lectures	2	30	participation	**3**
Singularities in Differential Topology	lectures	2	30	exam	**4**
Tutorials	tutorial	2	60	as arranged with tutor	**4**
Diploma project	individual work	10	300	diploma exam	**30**

Semi-algebraic Geometry. The course will start with the Tarski-Seidenberg Theorem with a proof of Łojasiewicz based on Thom's lemma. This proof gives immediately cell decompositions. It will be shown how to derive closedness of the class of semialgebraic sets with respect to topological operations. Next topics will be: stratifications, triangulations, curve selecting lemma.
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Singularities of Differentiable Mappings. The course will include the following topics: singular points of differential mappings, Sard's lemma, Thom's transversality theorem, Morse's lemma, Weierstrass preparation theorem for formal and convergent power series, the Malgrange-Mather preparation theorem, problem of classification of singularities, Whitney's classification theorem for transformations of the plane, Thom's elementary catastrophes.
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Semi- and Subanalytic Geometry. This approach to semi- and subanalytic geometry will be based on three fundamental tools: the Weierstrass preparation theorem for germs of analytic functions, a version of the Tarski- Seidenberg theorem for partially semialgebraic sets and the operation of blowing-up. It will be shown that classes of semi- and subanalytic sets are stable under topological operations. Next topics will be Gabrielov's theorem, Łojasiewicz's inequality and stratifications.
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Characteristic Classes. The basic construction of the Chern-Weil construction of characteristic classes for principal fiber bundles using invariant polynomials and curvature forms of connections. In this way the Pontrjagin and Chern classes are introduced. It will be checked that these classes satisfy the axioms of characteristic classes. The next class to be discussed is the Euler class. Finally, we will discuss characteristic classes as obstructions to the existence of certain geometric structures. At the end secondary characteristic classes and their variations will be discussed.
Responsible: Assoc. Prof. Robert Wolak

O-minimal Structures. This course will present the theory of o-minimal structures understood as a generalization of semialgebraic and subanalytic geometries. Only o-minimal structures on the field of real numbers will be considered. After presenting fundamental theorems such as : the monotonicity theorem and the cell decompo-sition theorem, the stress will be put on discussing examples of o-minimal structures, including o-minimal structure generated by all subanalytic functions and the exponential function.
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Extension Problems for Differentiable Functions. The course will start with the classical Whitney's extension theorem, for Whitney fields. Then the more delicate problem of extending C^k real functions will be considered, including Whitney's characterization of C^k - functions on closed subsets of the line. Special attention will be paid to the differentiable functions on sets with singularities, e.g. subanalytic sets.
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Singularities in Differential Topology. Smooth actions of compact groups, the structure of the orbit space. Singular foliations defined by Lie algebroids, Poisson manifolds, Poisson singularities, local description. Momentum map, singular symplectic reduction.
Responsible: Assoc. Prof. Robert Wolak

Algebraic Geometry. Affine and projective varieties, global properties: definition of projective and affine algebraic sets, ideal of an algebraic set, Hilbert Nullestellensatz, coordinate ring, decomposition, irreducible components, Zariski topology, regular and rational maps, dimension of an affine and projective variety; Local properties of affine varieties: local ring of a variety, normal varieties, tangent space (intrinsic nature), regular and singular points; Schemes and abstract varieties: sheaves, ringed spaces, spectrum of a ring, schemes, Proj of a graded ring, abstract varieties, morphisms of schemes, divisors and vector bundles; Cohomology: cohomology of sheaves, examples.
Responsible: Assoc. Prof. Robert Wolak